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| **Method** | **Input** | **Iteration** | **Idea behind method** | **Required for convergence** | **Pros** | **Cons** |
| Bisection | - Interval [a,b]  - Tolerance  - Maximum number of iteration | - Divide the interval [a, b] by two. Check the sign change and update new interval. | - Find the midpoint in the interval where the function changes the sign till finding the root | - The function is continuous.  - | - It is guaranteed that it converges.  - It is simple and reliable | - Slow convergence |
| Fixed point | - Initial guess (x\_0)  - Tolerance  - Maximum number of iteration  - A function x = g(x) | - | - Rearrange to , then find the iteration from till finding the root | - The function is continuous.  - The function is a contraction mapping on the interval (The absolute value of the first derivative at a fixed point is less than 1) | - Do not required the derivative function  - Can apply to more complicated functions | - Slow convergence if the first derivative at a fixed point is 1  - There are multiple ways to find g(x). It is difficult to find the most efficient g(x) |
| Newton | - Initial guess (x\_0)  - Tolerance  - Maximum number of iteration  - A function f(x) and the f’(x) | - | - Same as Fixed point, but | - Initial guess has to be close to the root to converge quadratically.  - The function is continuous and differentiable.  - The first derivative of the function is not zero in the interval. | - Fast convergence | - Required to find the derivative of the function  - Need a good initial guess to converge efficiently. |
| Secant | - Initial guess (x\_0 and x\_1 to find the tangent line)  - Tolerance  - Maximum number of iteration  - A function f(x) | - | - Same as Newton, but | - The function is continuous.  - Initial guess has to be close to the root to converge super linearly. | - Faster convergence than bisection method but slower than Newton  - Do not required the derivative function | - Need 2 initial guesses  - Converge slower than Newton (with an order close to the golden ratio) |